TRINITY COLLEGE

Mathematics Applications Units 3, 4
Test 5 2018

Calculator Assumed

Spanning Trees and Maximum Flow

STUDENT'S NAME Solutions

DATE: Thursday 16 August **TIME:** 45 minutes **MARKS**: 41

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

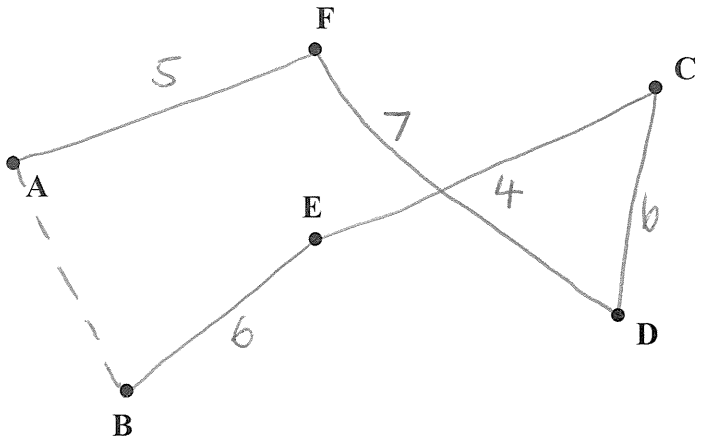
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (8 marks)

A water treatment plant has six storage tanks which need to be connected by pipelines. The estimated cost, in thousands of dollars, of installing a pipeline between any two tanks is given. The dash entries indicate that a pipeline cannot be installed.

	A	B	C	D	E	F
A	-	7	-	-	8	5
B	7	-	11	-	6	-
C	-	11	-	6	4	-
D	-	-	6	-	10	7
E	8	6	4	10	-	-
F	5	-	-	7	-	-

(a) Draw the minimal cost spanning tree. [4]



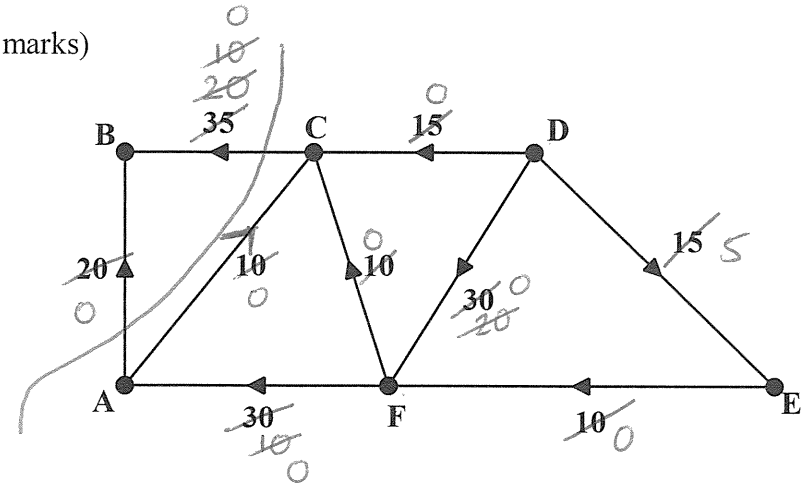
(b) Calculate the estimated minimum cost of connecting these tanks. [2]

$ 28,000

(c) It was discovered that the connection between tanks A and E could not proceed. What effect, if any, would this have on the estimated minimum cost? Explain. [2]

No effect, as AE is not used in the minimum spanning tree

2. (8 marks)



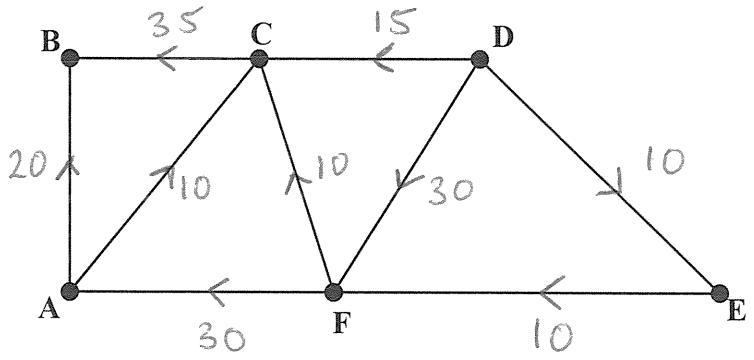
(a) Identify the source and the sink for the flow network above and label these on the network above. [2]

D: source B: sink

(b) Using the above diagram, determine the maximum flow through the network. Show the direction of flow in arc AC to achieve this. Show all systematic working and paths. [3]

DCB 15
DFCB 10
DFAB 20
DEFACB 10
55

(c) Draw a diagram showing the maximum flow and the direction of flow for each arc below. [2]

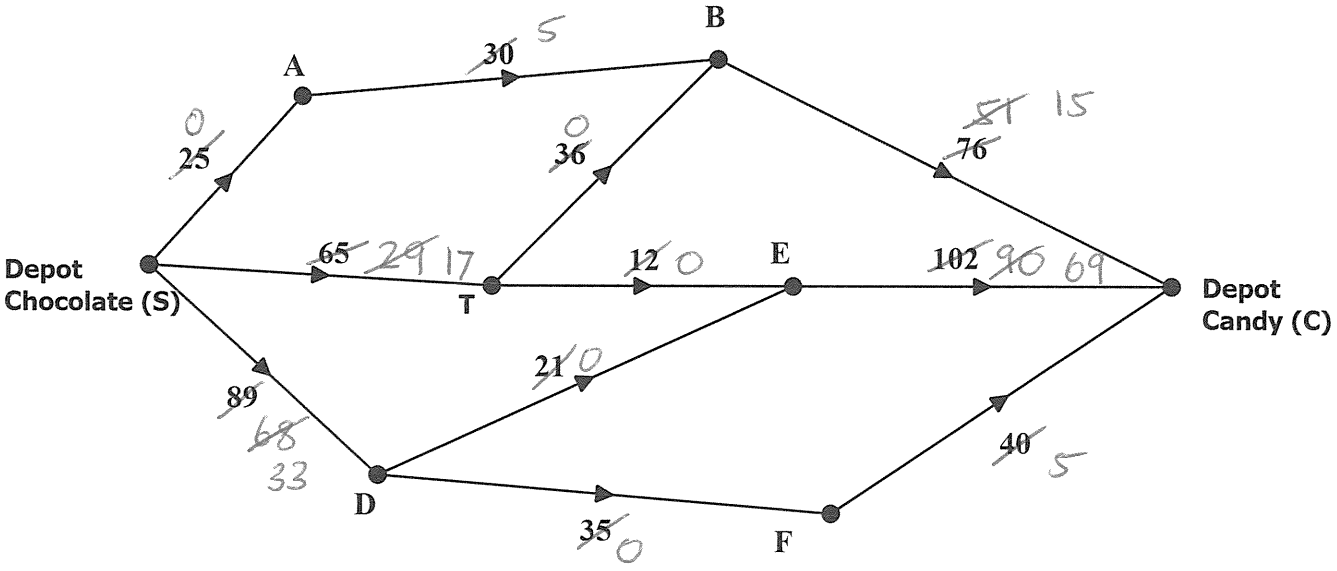


(d) Verify the value of the maximum flow by calculating a minimum cut and showing this cut on the above diagram. [1]

35 + 20 = 55

3. (8 marks)

The Oompa Loompas are busy at work in Willy Wonka's Chocolate Factory hauling kilograms of confectionery. The network below represents the "behind the scenes" Chocolate Factory paths and the amount of confectionery in kilograms that can be transported from Depot Chocolate (S) to Depot Candy (C) per hour. Each of the nodes represents a chocolate refuelling junction.



(a) Determine the maximum number of kilograms of confectionery that can be transported by the Oompa Loompas each hour from S to C. [3]

SABC 25
STBC 36
STEC 12
SDEC 21
SDFC 35

= 129 kg/h

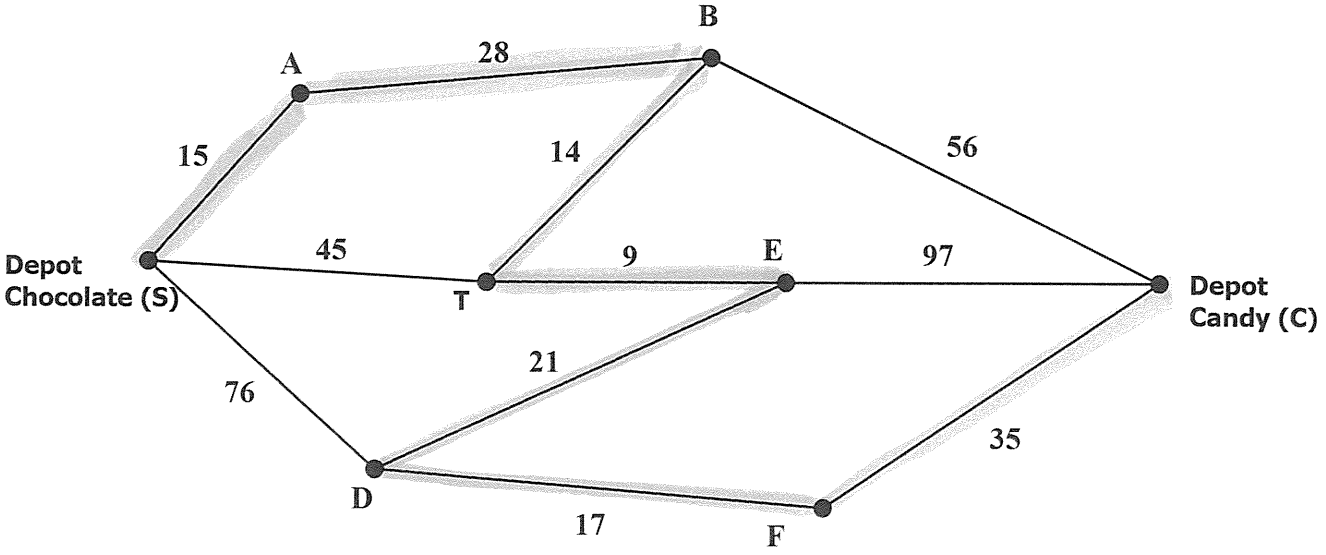
(b) Augustus Gloop drinks too much chocolate and gets sucked into path DF which consequently blocks the flow of confectionery transportation along this route. By how much does this closure of arc DF affect the maximum flow? [1]

Reduces flow by 35 kg/h

(c) Willy Wonka wants to improve the transportation of confectionery by assigning extra Oompa Loompas to just one arc. Using the original network, which arc should he choose in order to maximise the flow and what will be the new value of this arc? [2]

DE. Value is now 54 kg/h

Willy Wonka's security system involves lining each of the arcs between the depots with nerd rope. The network below represents the length of nerd rope required.



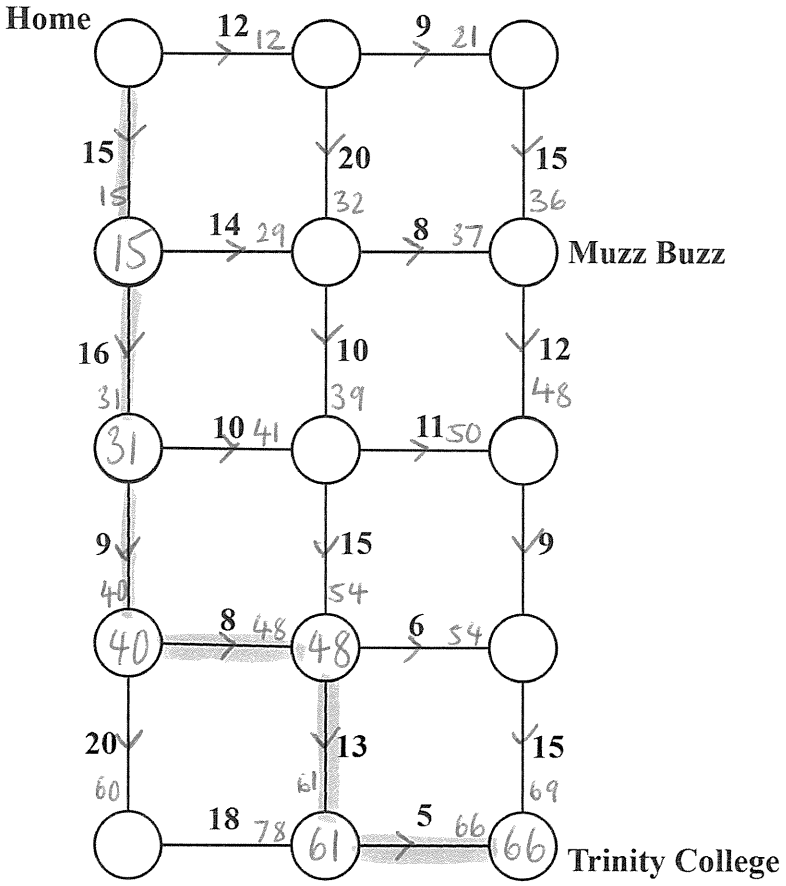
(d) If each depot is connected to each other depot (not necessarily directly), determine the minimum length of nerd rope needed for Willy Wonka's security system. Clearly indicate on the above network which arcs will be lined with nerd rope as part of the security system. [2]

$= 15 + 28 + 14 + 9 + 21 + 17 + 35$

$= 139$ units

4. (10 marks)

There are several alternate routes from Mr Presser's home to Trinity College. Distance is not a factor however, time delays at traffic lights and stop signs create the most annoyance. Out of frustration Mr Presser decides to time the delays in order to find the least frustrating way to drive to Trinity College. The time delays, in seconds, that he encounters on different routes are recorded on the network below. The paths through the network move either right or down.



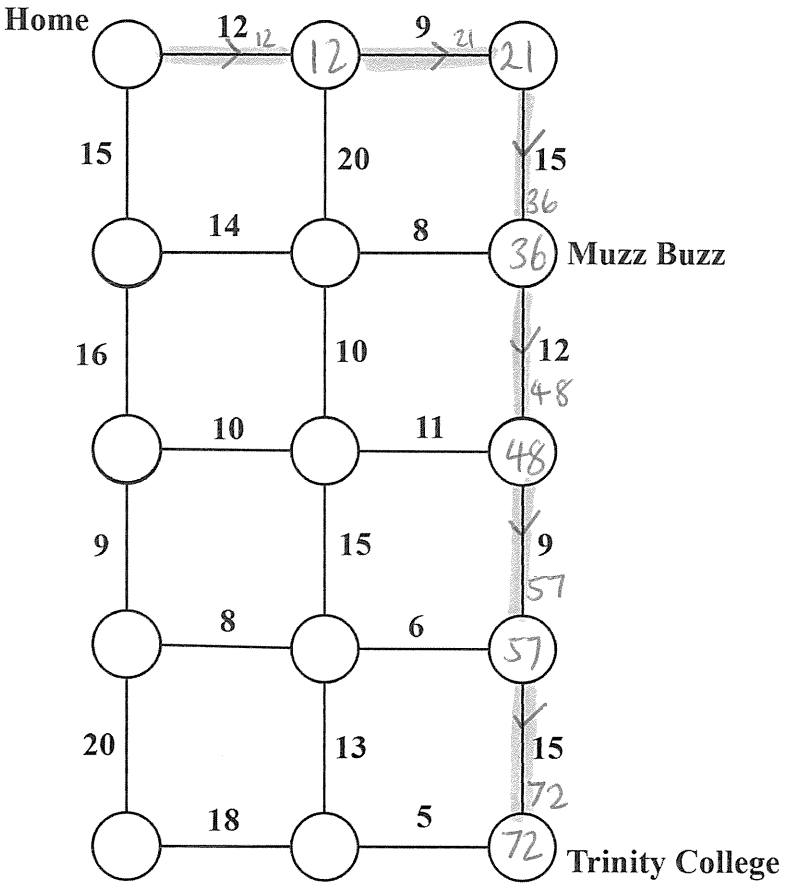
(a) On the network above, show the route that has the smallest total time delay from home to Trinity College. [2]

(b) What is this total smallest time delay? [1]

66 seconds

(c) To travel between each traffic light or stop sign it takes approximately 1 minute 15 seconds. On a particular day Mr Presser stops at Muzz Buzz for coffee and this takes 135 seconds.

(i) Show, on the second network below, the new route with the smallest time delay that Mr Presser should take to school if he stops at Muzz Buzz. [2]



(ii) What is this new total smallest time delay now? .3'

= 6(75) + 135 + 72 seconds
= 657 seconds

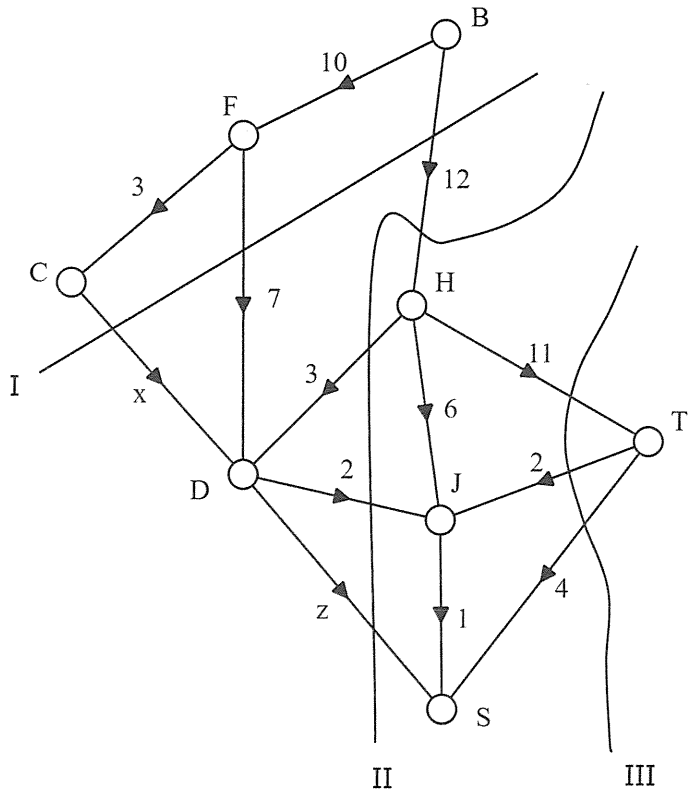
(iii) Did Mr Presser make a meeting that was scheduled for 7.30 am if he left home at 7.15 am? (Show all working to support your answer). .2'

72 + 6(75) + 135 = 657 seconds
= 10 mins, 57 seconds

∴ less than 15 mins so Mr Presser does make the meeting

5. (7 marks)

(a) Determine an expression or calculate the values of each of the following three cuts where possible. [3]

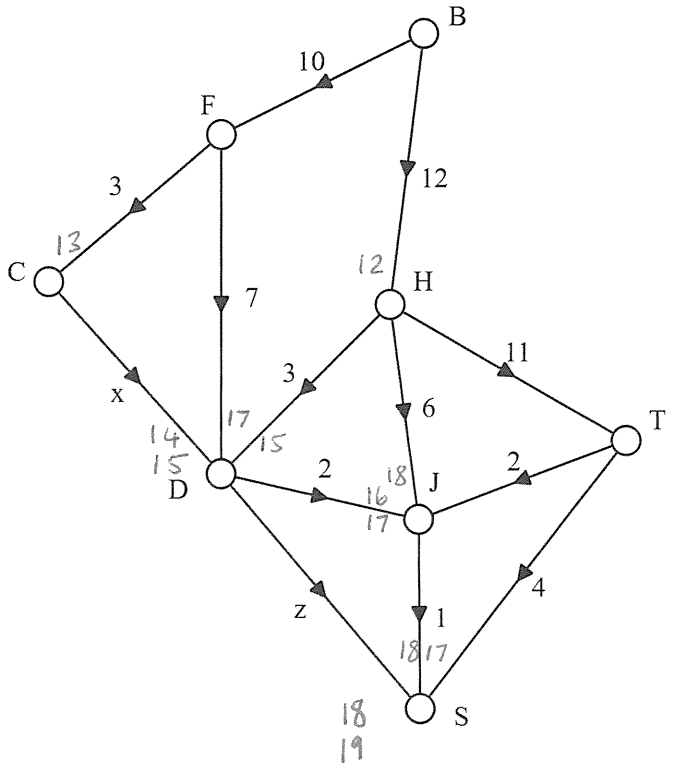


I: $19 + x$

II: $14 + z$

III: not a valid cut

(b) The shortest path from B to S in the network below is B – F – C – D – J – S.
All values are integers.



(i) Determine the possible positive integer value(s) of x ($x > 0$) which will achieve the shortest path B – F – C – D – J – S. [2]

$x = 1$ or 2

(ii) For the value(s) of x chosen in part (i), determine the minimum integer value that z can take to maintain the shortest path outline in part (i). [2]

$z = 3$ or $z \geq 4$